

- N.B. (1) Question No.1 is compulsory.  
 (2) Attempt any three questions out of the remaining five questions.  
 (3) Figures to right indicate full marks.

- Q1. (a) Evaluate  $\int_c |z| dz$ , where  $c$  is the left half of unit circle  $|z|=1$  from  $z=-i$  to  $z=i$  5
- (b) If  $\lambda$  is an Eigen value of the matrix  $A$  with corresponding Eigen vector  $X$ , prove that  $\lambda^n$  is an Eigen value of  $A^n$  with corresponding Eigen vector  $X$ . 5
- (c) Find the extremal of  $\int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{x} dx$  5
- (d) Find the unit vector orthogonal to both  $[1,1,0]$  &  $[0,1,1]$  5
- Q2. (a) Find the curve on which the functional  $\int_0^1 [y'^2 + 12xy] dx$  with  $y(0)=0$  &  $y(1)=1$  can be Extremised. 6
- (b) Find the Eigen values and Eigen vectors for the matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$  6
- (c) Obtain two distinct Laurent's series expansions of  $f(z) = \frac{2z-3}{z^2-4z+3}$  in powers of  $(z-4)$  indicating the region of convergence in each case 8
- Q3. (a) If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ , find  $A^{50}$  6
- (b) Evaluate  $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ , where  $c$  is the circle  $|z|=3$  6
- (c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of the functional  $I(y) = \int_0^1 (y'^2 - 2y - 2xy) dx$  subject to  $y(0)=2$ ,  $y(1)=1$ . 8

Q4. (a) Find the vector orthogonal to both  $[-6, 4, 2]$  &  $[3, 1, 5]$  6

(b) Show that the matrix  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory

and find its minimal polynomial. 6

(c) Reduce the matrix of the quadratic form  $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$  to canonical form through congruent transformation and find its rank, signature, and value class. 8

Q5. (a) Find the extremal of  $\int_{x_0}^{x_1} (2xy - y''^2) dx$  6

(b) Show that the set  $W = \{[x, y, z] \mid y = x + z\}$  is a subspace of  $\mathbf{R}^3$  under the usual addition and scalar multiplication. 6

(c) Show that the following matrix  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  is diagonalisable. Also find the

diagonal form and a diagonalising matrix. 8

Q6. (a) If  $f(a) = \int_c \frac{3z^2 + 7z + 1}{z - a} dz$ , where  $c$  is a circle  $|z| = 2$ , find the values of

i)  $f(-3)$ , ii)  $f(i)$ , iii)  $f'(1-i)$   $\rightarrow f'(1-i)$  6

(b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$  6

(c) Verify Cayley-Hamilton theorem for the matrix  $A$  and hence find  $A^{-1}$  and  $A^4$ .

$$\text{Where } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

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